



**The total mark of this exam is: 40**

**This exam paper consists of 3 pages**

**Question 1:**

**[12 M]**

- An antenna has a uniform field  $E=2V/m$  (rms) at a distance of 100 m where:  
 $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}, 0 \leq \phi \leq \frac{\pi}{2}$  with  $E=0$  elsewhere, the antenna terminal current is 3 A (rms), suppose antenna operates at 1 GHz. Find: (Use:  $U_n = E_n = 1$ )
  - the directivity of the antenna,
  - effective aperture and
  - radiation resistance.
- Show that the directivity for a source with a unidirectional power pattern given by:  
 $U = U_m \cos^n \theta$  can be expressed as  $D = 2(n+1)$ .  $U$  has a value only for  
 $0 \leq \theta \leq \frac{\pi}{2}$ . The patterns are independent of the azimuth angle  $\phi$ .
- An 875 MHz signal is to be transmitted over a 1 Km distance using antenna with the gain of 5dBi (the transmitting and the receiving antennas are identical) the transmitter has 1 Watt of power available at the input terminals of the transmitter antenna which is perfectly matched. Find the signal strength (Power) at the terminal of the received antenna.

**Question 2:**

**[10 M]**

**For the following data:  $N = 3, d = \frac{\lambda}{3}, \delta = \frac{\pi}{2}$ , find the:**

- Angles (in degrees) where the nulls of the array factor occur.
- Angles (in degrees) where the maximum of the array factor occur.
- Angles (in degrees) where the power is half of its maximum.
- Angles (in degrees) where the side lobes maxima occur.
- Draw the radiation pattern for the array factor.

**Question 3:**

**[10 M]**

Estimate the relative field pattern (equation) of an array of two identical isotropic point sources along the z axis. where  $d = \lambda, \delta = \pi$

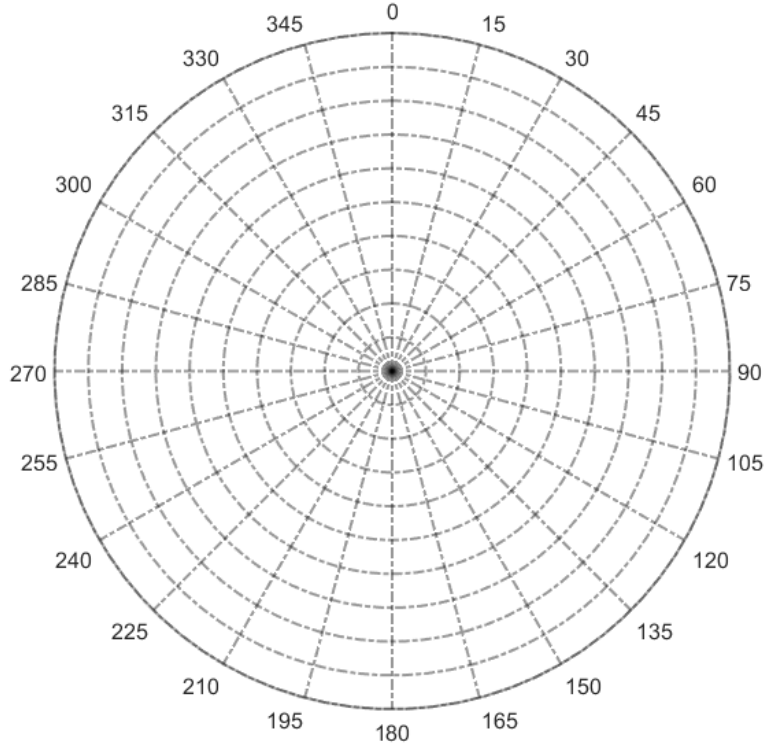
**Question 4:**

**[8 M]**

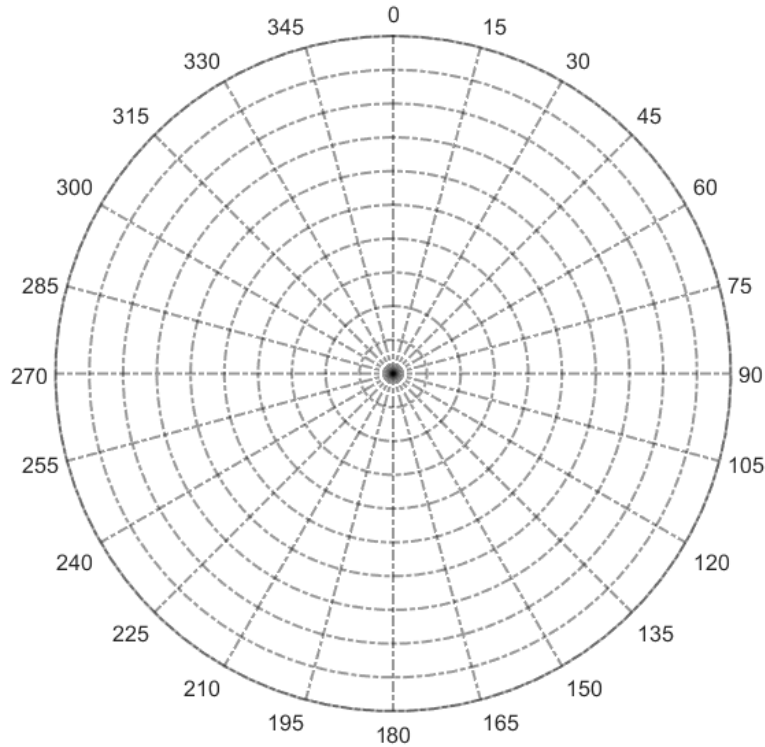
Design a *rectangular* microstrip antenna so that it will resonate at 2 GHz. The idealistic lossless substrate (RT/Duroid 6010.2) has a dielectric constant of 10.2 and a height of 0.127 cm. Determine:

- The practical width that leads to good radiation efficiency.
- The effective dielectric constant of the patch.
- The effective length of the patch.
- The actual length of the patch.

**Radiation pattern for Question 2:**



**Radiation pattern for Question 3:**



Parameter	Formula	Parameter	Formula
Infinitesimal area of sphere	$dA = r^2 \sin \theta d\theta d\phi$	Elemental solid angle of sphere	$d\Omega = \sin \theta d\theta d\phi$
Average power density	$\mathbf{W}_{av} = \frac{1}{2} \text{Re}[\mathbf{E} \times \mathbf{H}^*]$	Radiated power/average radiated power	$P_{rad} = \iint_S \mathbf{W}_{av} \cdot d\mathbf{s} = \frac{1}{2} \iint_S \text{Re}[\mathbf{E} \times \mathbf{H}^*] \cdot d\mathbf{s}$
Radiation density	$W_{av} = \frac{P_{rad}}{4\pi r^2}$	Radiation intensity (far field)	$U = r^2 W_{av}$
Directivity $D(\theta, \phi)$	$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} = \frac{4\pi}{\Omega_A}$	Beam solid angle $\Omega_A$	$\Omega_A = \int_0^{2\pi} \int_0^\pi U_n(\theta, \phi) \sin \theta d\theta d\phi$ $U_n(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{max}}$
Gain $G(\theta, \phi)$	$G = \frac{4\pi U(\theta, \phi)}{P_{in}} = e_{cd} \left[ \frac{4\pi U(\theta, \phi)}{P_{rad}} \right] = e_{cd} D(\theta, \phi)$	Antenna radiation efficiency $e_{cd}$	$P_{rad} = e_{cd} P_{in}$ $e_{cd} = \frac{R_r}{R_r + R_L}$
Maximum effective area $A_{em}$	$A_{em} = e_{cd} \left( \frac{\lambda^2}{4\pi} \right) D = \left( \frac{\lambda^2}{4\pi} \right) G$	Friis transmission equation	$\frac{P_r}{P} = \left( \frac{\lambda}{4\pi R} \right)^2 G_t G_r$
Aperture efficiency $\epsilon_{ap}$	$\epsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}$	Wave impedance $Z_w$	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377 \text{ ohms}$

Parameter	Formula	Parameter	Formula
	<b>Infinitesimal Dipole</b> ( $l \leq \lambda/50$ )		<b>Small Dipole</b> ( $\lambda/50 < l \leq \lambda/10$ )
Radiation resistance $R_r$	$R_r = \eta \left( \frac{2\pi}{3} \right) \left( \frac{l}{\lambda} \right)^2 = 80\pi^2 \left( \frac{l}{\lambda} \right)^2$	Radiation resistance $R_r$	$R_r = 20\pi^2 \left( \frac{l}{\lambda} \right)^2$
Half-power beamwidth	HPBW = 90°	Half-power beamwidth	HPBW = 90°
	<b>Half Wavelength Dipole</b> ( $l = \lambda/2$ )		<b>Finite length Dipole</b> ( $l > \lambda/10$ )
Radiation resistance $R_r$	$R_r = \frac{\eta}{4\pi} C_{in}(2\pi) \simeq 73 \text{ ohms}$	Electric field intensity $E_\theta$	$E_\theta \simeq j\eta \frac{I_0 e^{j\beta r}}{2\pi r} \left[ \frac{\cos\left(\frac{\beta l}{2} \cos \theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin \theta} \right]$
Half-power beamwidth	HPBW = 78°		

2 Isotropic Sources with Same amplitude with any phase difference		N-ELEMENT LINEAR ARRAY: UNIFORM AMPLITUDE AND SPACING	
ARRAY FACTOR	$(E)_n = \cos\left[\frac{1}{2}(\beta d \cos \theta + \delta)\right]$		where $\psi = \beta d \cos \theta + \delta$
<b>Patch Microstrip Antenna</b>			$(E)_n \simeq \left[ \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} \right]$
	$W = \frac{c}{2f_r} \sqrt{\frac{2}{\epsilon_r + 1}}$		● <b>Nulls</b>
	$\epsilon_{reff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + \frac{12h}{W} \right]^{-1/2}$	ARRAY FACTOR	$\sin\left(\frac{N}{2}\psi\right) = 0 \Rightarrow \frac{N}{2}\psi_{\theta=\theta_n} = \pm n\pi$ $n = 1, 2, 3, \dots$ $n \neq N, 2N, 3N, \dots$
	$\frac{\Delta L}{h} = 0.412 \frac{(\epsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\epsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)}$		● <b>Maximum</b>
	$L_{eff} = \frac{c}{2f_r} \sqrt{\frac{1}{\epsilon_{reff}}}$		$\psi = 0$
	$L = L_{eff} - 2\Delta L$		● <b>3- dB point</b>
			$\frac{N}{2}\psi = \frac{N}{2}(\beta d \cos \theta + \delta) _{\theta=\theta_n} = \pm 1.391$
			● <b>Side lobes Maxima</b>
			$\frac{N}{2}\psi = \pm \frac{(2s+1)}{2} \pi$ $s = 1, 2, 3, \dots$

①

② i  $E = 2V/m$ ,  $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$ ,  $0 \leq \phi \leq \frac{\pi}{2}$   
 $r = 100m$ ,  $I = 3A$ ,  $f = 1GHz$   
rms

a)  $D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin(\theta) d\theta \int_0^{\frac{\pi}{2}} d\phi} = \frac{4\pi}{\left[-\cos(\theta)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left[\phi\right]_0^{\frac{\pi}{2}}}$  ②

$= \frac{4\pi}{\frac{\pi}{2} \left[\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{3}\right)\right]} = \frac{8}{\frac{\sqrt{3}}{2} - \frac{1}{2}} = \frac{16}{\sqrt{3} - 1} = 21.856$

b)  $A_e = \frac{\lambda^2}{4\pi} (D) = \frac{\lambda^2}{4\pi} (21.856) = 1.139 \lambda^2 = 1.739 \left(\frac{3 \times 10^8}{1 \times 10^9}\right)^2$

$= 0.1565 m^2$  ②

c)  $R_r \Rightarrow \frac{1}{2} \int_0^2 R_r = P_r = 9R_r$

$P_r = \int_S W_{av} \cdot r^2 \sin(\theta) d\theta d\phi$

$\Rightarrow W_{av} = \frac{1}{2} R_e [E \cdot H^*] = \frac{1}{2} \frac{E^2}{\eta} = \frac{1}{2} \frac{(2)^2}{120\pi} = \frac{1}{60\pi}$

$P_r = \iint_S \frac{1}{60\pi} (100)^2 \cdot \sin(\theta) d\theta d\phi$

$= 53.05 \int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin(\theta) d\theta d\phi = 53.05 (0.575)$   
 $= 30.5W$

$P_r = 30.5W = 9R_r \Rightarrow R_r = 3.389 \Omega$  ②

(2)

(ii)  $U = U_m \cos^n(\theta)$

$$D = \frac{4\pi}{\int_s U_n \sin(\theta) d\theta d\phi}$$

$$U_n = \cos^n(\theta)$$

$$D = \frac{4\pi}{\int_0^{\pi/2} \cos^n(\theta) \sin(\theta) d\theta \int_0^{2\pi} d\phi} = \frac{4\pi}{2\pi \left[ -\frac{\cos^{n+1}(\theta)}{n+1} \right]_0^{\pi/2}}$$

$$= \frac{2}{-\left[0 - \frac{1}{n+1}\right]} = \frac{2}{\left[\frac{1}{n+1}\right]} = 2(n+1)$$

(iii)  $f = 875 \text{ MHz}$ ,  $R = 1 \times 10^3 \text{ m}$ ,  $G_t = G_r = 5 \text{ dB}_i = 3.162$   
 $P_t = 1 \text{ W}$        $P_r = ?$

$$\lambda = \frac{3 \times 10^8}{875 \times 10^6} = 0.343 \text{ m}$$

$$P_r = \left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r$$

$$P_r = (0.000273)^2 (3.162) = 7.7489 \times 10^{-9} \text{ W} = 7.7489 \text{ nW}$$
  
 $= -81.106 \text{ dBW}$

(3)

Q2 :-  $N=3$ ,  $d=\frac{\lambda}{3}$ ,  $\delta=\frac{\pi}{2}$

For max  $\psi=0 \Rightarrow \beta d \cos(\theta_m) + \delta = 0$   $\beta d = \frac{2\pi}{\lambda} \left(\frac{\lambda}{3}\right) = \frac{2\pi}{3}$

$$\frac{2\pi}{3} \cos(\theta_m) + \frac{\pi}{2} = 0 \Rightarrow \cos(\theta_m) = -\frac{3}{4}$$

$$\theta_m = \pm 138.59^\circ \Rightarrow 138.59^\circ, 221.41^\circ$$

For Nulls :-  $\frac{N}{2} \psi = \pm n\pi$

$$\frac{3}{2} \psi = \pm n\pi \Rightarrow \psi = \pm \frac{2}{3} n\pi$$

$$\frac{3}{2} \left( \frac{2\pi}{3} \cos(\theta_n) + \frac{\pi}{2} \right) = \pm n\pi$$

$$\frac{2}{3} \cos(\theta_n) + \frac{1}{2} = \pm \frac{2}{3} n$$

$$\cos(\theta_n) = \frac{3}{2} \left( -\frac{1}{2} \pm \frac{2}{3} n \right)$$

for  $n=1 \Rightarrow \cos(\theta_n) = \frac{3}{2} \left( -\frac{1}{2} \pm \frac{2}{3} \right) \rightarrow \frac{1}{4} \Rightarrow \pm 75.5^\circ$   
 $n=2, 3, 4$ , rejected  $\rightarrow 1.755$  rejected.

$$\theta_n = 75.5^\circ, 284.5^\circ$$

For sidelobes :-  $\frac{N\psi}{2} = 1 \Rightarrow \frac{3\psi}{2} = \pm \frac{(2s+1)\pi}{2}$

$$\psi = \frac{(2s+1)\pi}{3} \Rightarrow \frac{2\pi}{3} \cos(\theta_s) + \frac{\pi}{2} = \pm \frac{(2s+1)\pi}{3}$$

$$\cos(\theta_s) = \frac{3}{2} \left( -\frac{1}{2} \pm \frac{(2s+1)\pi}{\pi} \right)$$

for  $s=0 \Rightarrow \theta_s = \pm 104.47^\circ \Rightarrow$  rejected within the main lobe

for  $s=1 \Rightarrow \cos(\theta_s) = \frac{3}{2} \left( -\frac{1}{2} \pm 1 \right) \Rightarrow \cos(\theta_s) = \pm 0.75$

$$\theta_s = 41.409^\circ, 318.59^\circ$$

(4)

Half power: -  $\frac{NV}{2} = \pm 1.391$

$$\frac{3}{2} \psi = \pm 1.391$$

$$\psi = \pm \frac{2}{3} (1.391) = \pm 0.9273$$

$$\frac{2\pi}{3} \cos(\theta_n) + \frac{\pi}{2} = \pm 0.9273$$

$$\frac{2}{3} \cos(\theta_n) + \frac{1}{2} = \pm 0.2952$$

$$\cos(\theta_n) = \frac{3}{2} \left( -\frac{1}{2} \pm 0.2952 \right)$$

$$\cos(\theta_n) = -0.3072 \Rightarrow \theta_n \approx \pm 107.89^\circ$$

$$\theta_n = 107.89^\circ, 252.11^\circ \quad (2)$$

2 marks for radiation pattern.

Q3:-

(5)

$$d = \lambda \quad \delta = \pi$$

$$E_n = \frac{\cos\left(\frac{\beta d \cos\theta + \delta}{2}\right)}{\cos\left(\frac{2\pi x}{2\lambda} \cos\theta + \frac{\pi}{2}\right)}$$

$$E_n = \cos\left(\pi \cos(\theta) + \frac{\pi}{2}\right)$$

For max  $E_n = \pm 1$

$$\pi \cos(\theta_m) + \frac{\pi}{2} = \pm n\pi$$

$$\cos(\theta_m) = \frac{-1 \pm n}{2}$$

for  $n=0 \Rightarrow \cos(\theta_m) = -\frac{1}{2} \Rightarrow \theta_m = \pm 120^\circ \Rightarrow 120^\circ, 240^\circ$  (2)

for  $n=1 \Rightarrow \cos(\theta_m) = \frac{-1 \pm 1}{2} \rightarrow -\frac{3}{2} \Rightarrow$  rejected

$\rightarrow \frac{1}{2} \Rightarrow \theta_m = 60^\circ, 300^\circ$

For nulls:  $E_n = 0$

$$\pi \cos(\theta_m) + \frac{\pi}{2} = \pm (2n+1) \frac{\pi}{2}$$

$$\cos(\theta_n) = \frac{-1 \pm (2n+1)}{2}$$

for  $n=0$  :-  $\cos(\theta_n) = \frac{-1 \pm 1}{2} \rightarrow -1 \Rightarrow \theta_n = 180^\circ$   
 $\rightarrow 0 \Rightarrow \theta_n = 90^\circ, 270^\circ$  (2)

for  $n=1$  :-  $\cos(\theta_n) = \frac{-1 \pm 3}{2} \rightarrow -2 \Rightarrow$  rejected  
 $\rightarrow 1 \Rightarrow \theta_n = 0^\circ$

For half points  $\Rightarrow E_n = \pm \frac{1}{\sqrt{2}}$

$$\pi \cos(\theta_n) + \frac{\pi}{2} = \pm (2n+1) \frac{\pi}{4}$$

$$\cos(\theta_n) = \frac{-1 \pm (2n+1)}{4}$$

for  $n=0 \Rightarrow \cos(\theta_n) = \frac{-1 \pm 1}{4} \rightarrow -\frac{3}{4} \Rightarrow \theta_n = 138.6^\circ, 221.4^\circ$  (2)  
 $\rightarrow -\frac{1}{4} \Rightarrow \theta_n = 104.47^\circ, 255.52^\circ$



(6)

For  $n=1 \Rightarrow \cos(\theta_n) = -\frac{1}{2} \pm \frac{3}{4} \rightarrow \frac{5}{4} \Rightarrow$  rejected  
 $\rightarrow \frac{1}{4} \Rightarrow 75.52^\circ, 284.47^\circ$

For  $n=2 \Rightarrow \cos(\theta_n) = -\frac{1}{2} \pm \frac{5}{4} \rightarrow \frac{3}{4} \Rightarrow$  rejected  
 $\rightarrow \frac{7}{4} \Rightarrow 41.41^\circ, 318.6^\circ$

Q4: -  $F=2\text{GHz}$ ,  $\epsilon_r=10.2$ ,  $h=0.127\text{ cm}$

+2 for radiation pattern

$$W = \frac{3 \times 10^8}{2 \times 2 \times 10^9} \sqrt{\frac{2}{10.2 + 1}} = \frac{0.3}{4} \sqrt{\frac{2}{11.2}} = 3.169\text{ cm}$$

$$\epsilon_{\text{reff}} = \frac{11.2}{2} + \frac{9.2}{2} \left( \frac{1 + 12(0.127)}{3.169} \right) = 5.6 + \frac{4.6}{\sqrt{1.481}}$$

$$= 5.6 + \frac{4.6}{1.217} = 5.6 + 3.78 = 9.38$$

$$\frac{\Delta L}{h} = 0.412 \frac{(9.38 + 0.3) \left( \frac{3.169}{0.127} + 0.264 \right)}{(9.38 - 0.258) \left( \frac{3.169}{0.127} + 0.8 \right)}$$

$$\frac{\Delta L}{0.127} = 0.412 \frac{(9.68) (24.953 + 0.264)}{(9.122) (24.953 + 0.8)}$$

$$\frac{\Delta L}{0.127} = 0.412 \frac{(9.68) (25.217)}{(9.122) (25.753)}$$

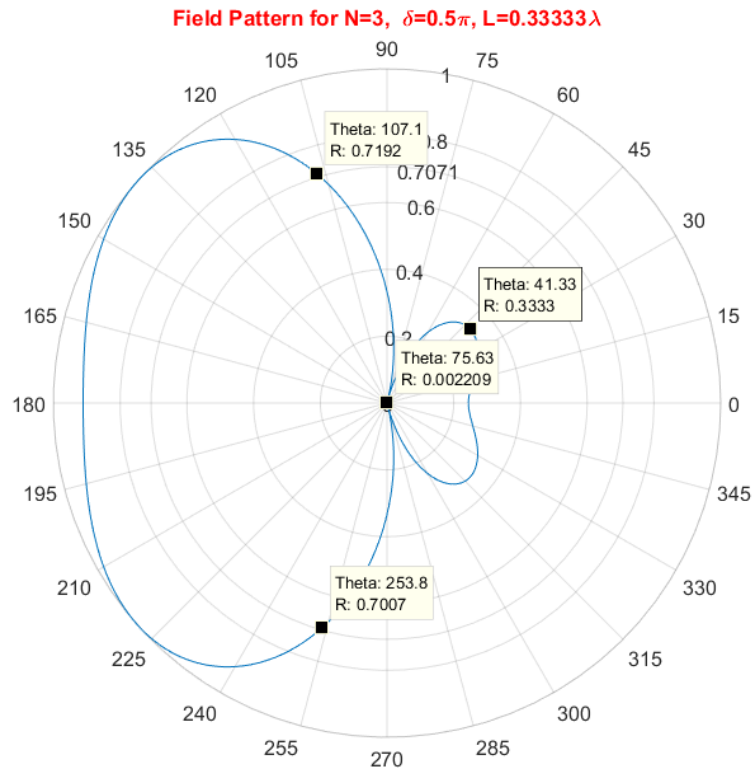
$$\Delta L = (0.127)(0.412) \frac{244.5}{234.92}$$

$$\Delta L = (0.0523)(1.0391) = 0.0543\text{ cm}$$

$$L_{\text{eff}} = \frac{3 \times 10^8}{2 \times 2 \times 10^9} \sqrt{\frac{1}{9.38}} = \frac{0.3}{4} (0.3265) = 2.448\text{ cm}$$

$$L = L_{\text{eff}} - 2\Delta L = 2.448 - 2(0.0543) = 2.34\text{ cm}$$

### Radiation pattern for Question 2:



### Radiation pattern for Question 3:

